Indian Statistical Institute Bangalore
Semester Examination
Date: May 02, 2019.

Topology B.Math II Instructor: Santhosh Kumar P Total : 50

Answer any five questions. Each question carries ten marks.

(1) Prove that $\Big\{\{x, n, n+1\} : x \in \mathbb{R} \;\text{ and }\; n \in \mathbb{N}\Big\}$ is a subbasis for $\mathbb{R}$ with discrete topology. Is $\mathbb{R}$ with lower limit topology a second countable space? Justify your answer. [5+5]

(2) Prove the following statements.
  (a) A locally compact Hausdorff space is completely regular. [5]
  (b) One point compactification of $\mathbb{R}^2$ is the 2 - sphere [5]

$$\mathbb{S}^2 = \{(x, y, z) \in \mathbb{R}^3 : \; x^2 + y^2 + z^2 = 1\}.$$

(3) Define regular space. Prove that if X is a compact Hausdorff space, then X is normal. Is X metrizable? Justify your answer. [10]

(4) Let (X, d) be a metric space. If X is connected, then X is chainable. Is the converse true ? Justify your answer. [10]

(5) Let X be a topological space and $A \subset X$ be a nonempty closed. Let $\widetilde{X}$ be the quotient space defined by (collapsing A to singleton) the following relation $x \sim y$ if and only if either $x, y \in A$ or $x = y$. Show that
  (a) if X is normal, then $\widetilde{X}$ is normal. [5]
  (b) If A is compact and X is locally compact metric space, then $\widetilde{X}$ is compact and metrizable. [5]

(6) Define quotient topology. Let $\mathbb{S}^1 = \{z \in \mathbb{C} : \; |z| = 1\}$ and the cone $c(\mathbb{S}^1)$ is defined as the quotient of $\mathbb{S}^1 \times [0, 1]$ obtained by collapsing $\mathbb{S}^1 \times \{0\}$ to singleton. Then show that $c(X)$ is homeomorphic to the closed unit disc [10]

$$\mathbb{D} = \{z \in \mathbb{C} : \; |z| \leq 1\}.$$

P. T. O

(7) Let X be a topological space and $\mathcal{S}$ be a subbasis for X. Show that X is compact if and only if for every collection of closed subsets $\{Z_\alpha : \alpha \in J\}$ of X having finite intersection property and satisfying $X \setminus Z_\alpha \in \mathcal{S}$ for each $\alpha \in J$, we have that

$$\bigcap_{\alpha \in J} Z_\alpha \neq \emptyset.$$

[10]

∗ ∗ ∗ ALL THE BEST ∗ ∗ ∗